



DRIVING  
THE EXASCALE  
TRANSITION



Institut Català  
de Nanociència  
i Nanotecnologia

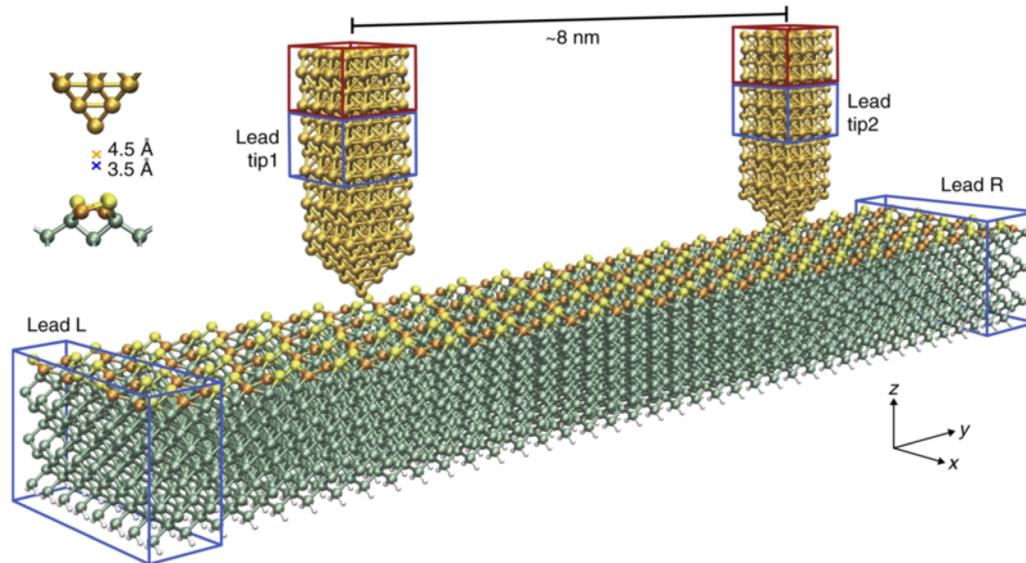


# HPC-enabled very large scale quantum simulations in materials with SIESTA

Pablo Ordejón (ICN2, Barcelona)



## Electronic transport in nanoscale devices

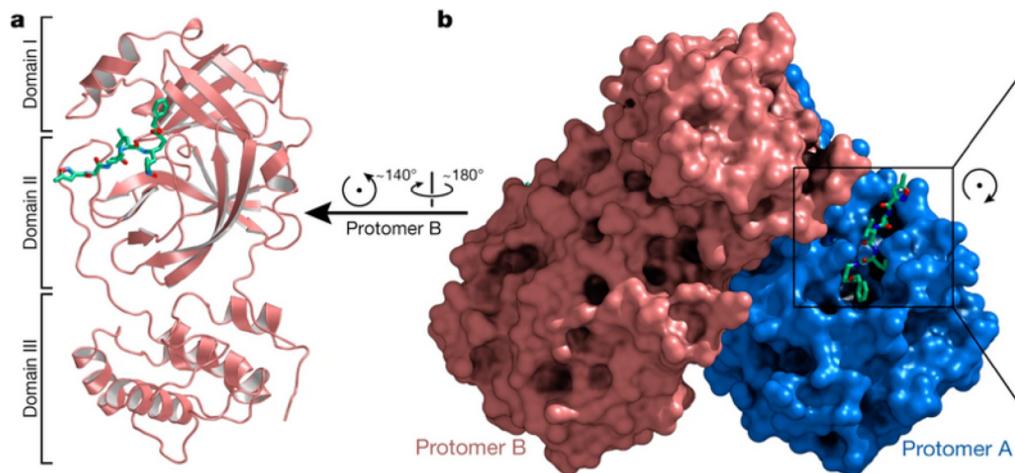


Four-terminal electron transport on Ge surface and gold tips

**4924 atoms**

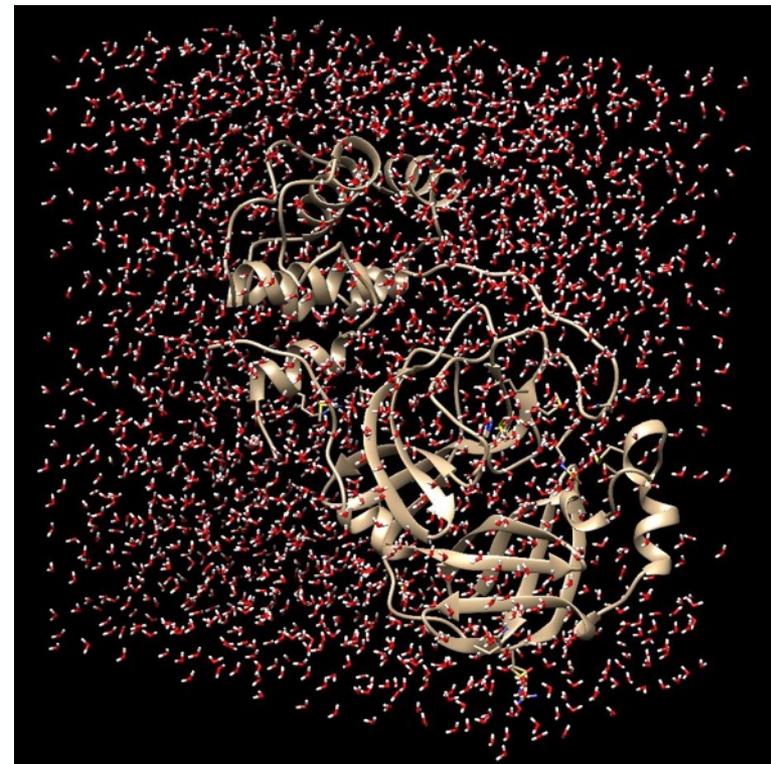
M. Kolmer et al Nature Commun. 2019

## SARS CoV-2 M<sup>pro</sup> - interaction with inhibitor drugs

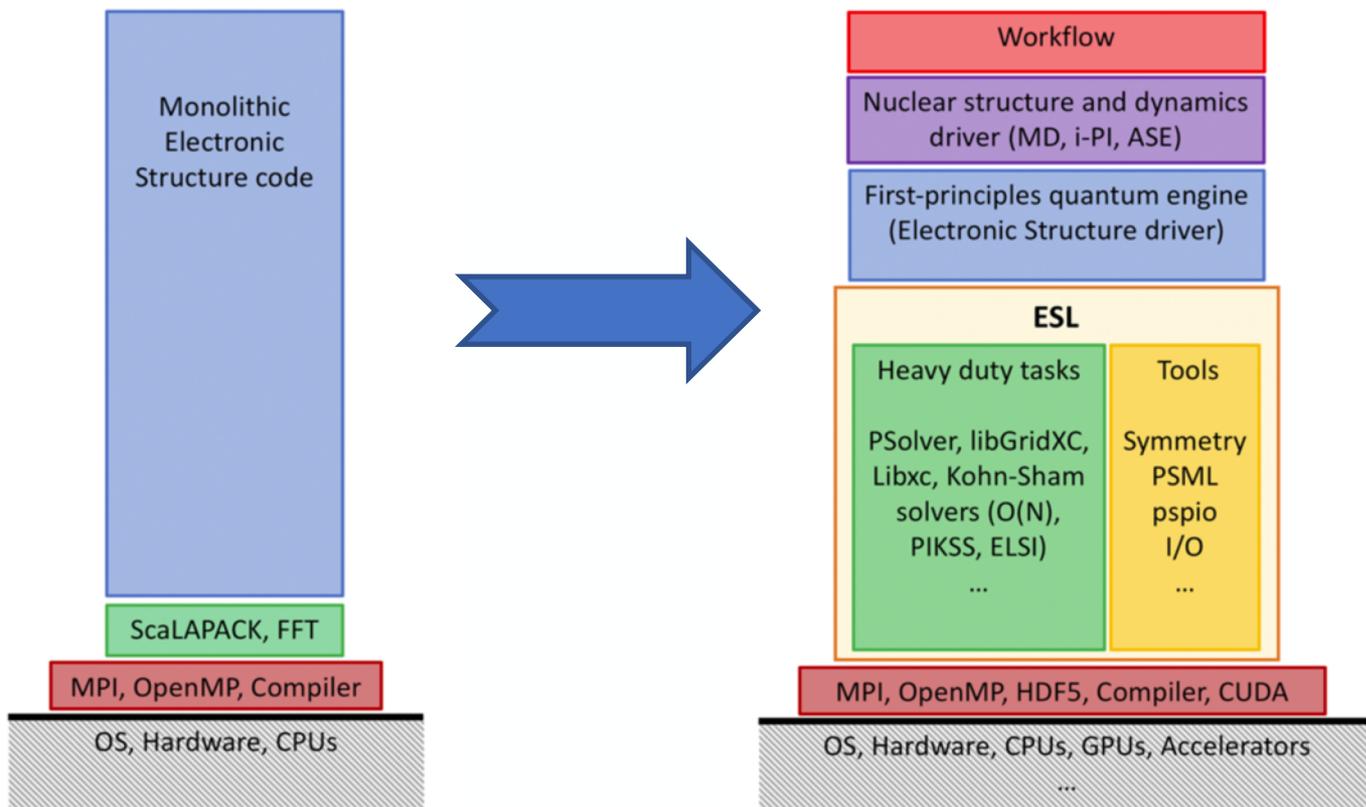


Jin et al, Nature 582, 289 (2020)

Protomer B in solvation  
**~8800 atoms**



## Changing paradigm



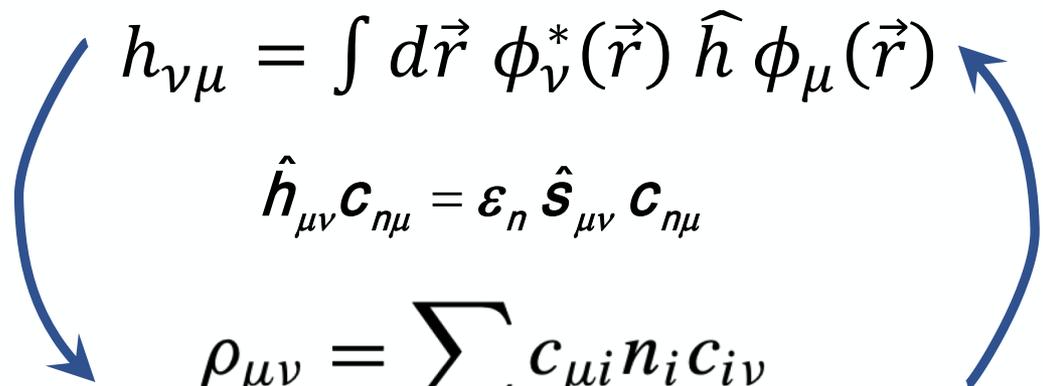
SIESTA is a DFT code, density-functional theory  
(like many others in many ways)

Aim from inception: **EFFICIENCY**

$$\psi_n(\mathbf{r}) = \sum_{\mu} c_{n\mu} \phi_{\mu}(\mathbf{r})$$

$$h_{\nu\mu} = \int d\vec{r} \phi_{\nu}^*(\vec{r}) \hat{h} \phi_{\mu}(\vec{r})$$

$$\hat{h}_{\mu\nu} \mathbf{c}_{n\mu} = \epsilon_n \hat{\mathbf{s}}_{\mu\nu} \mathbf{c}_{n\mu}$$

$$\rho_{\mu\nu} = \sum_i c_{\mu i} n_i c_{i\nu}$$


Pioneer **LINEAR-SCALING** DFT code (or Order-N,  $O(N)$  )

meaning: computational cost (CPU & memory) scaling linearly with  
number of atoms

# The Solvers

We need to obtain the density from the Hamiltonian

Direct eigenvalue/eigenvector solvers

Obtaining the density directly

Fermi-Dirac function

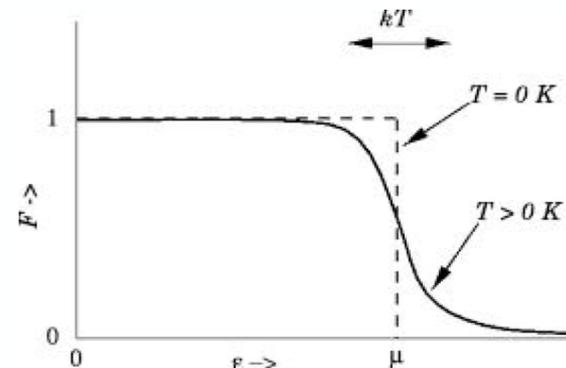
$$\hat{h}_{\mu\nu} \mathbf{c}_{n\mu} = \epsilon_n \hat{\mathbf{s}}_{\mu\nu} \mathbf{c}_{n\mu}$$

$$\hat{\rho} = f_{\beta}(\hat{H} - \mu)$$

$$\rho_{\mu\nu} = \sum_i c_{\mu i} n_i c_{i\nu}$$

$$f_{\beta}(\epsilon_i - \mu) = \frac{2}{1 + e^{\beta(\epsilon_i - \mu)}}$$

$$\rho(\mathbf{r}) = \sum_{\mu\nu} \rho_{\mu\nu} \phi_{\nu}^*(\mathbf{r}) \phi_{\mu}(\mathbf{r})$$



Need to use DIRECT solvers, as the number of desired eigenvectors is a substantial fraction of the matrix size

SIESTA uses pre-packaged libraries for this pure math problem:

- **ScaLaPACK**
  - pdsyev, pzheev and related drivers
  - MRRR
- **ELPA**: Alternative transformation sequence + optimizations  
<https://elpa.mpcdf.mpg.de/>
- **Conversion of H and S to dense form**
- Cholesky decomposition to reduce to standard eigenproblem
- Transformation to tri-diagonal form
- Solution of tri-diagonal problem
- Back-transformation

**Cubic scaling** with matrix size — Quadratic scaling for memory

**Method of choice for "small" problems  
(some hundreds of atoms)**

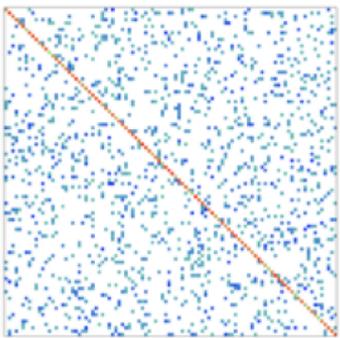
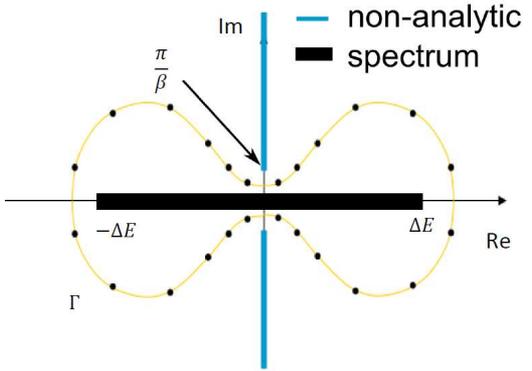
# Direct solution – Fermi Operator

## PEXSI: Pole Expansion plus **Selected Inversion**

(Lin Lin, Chao Yang, et al., Berkeley)

Lin, García, Huhs, Yang, JPCP 2014

$$f_\beta(\epsilon_i - \mu) = \frac{2}{1 + e^{\beta(\epsilon_i - \mu)}} \approx \text{Im} \sum_{l=1}^P \frac{\omega_l}{\epsilon_i - (z_l + \mu)}$$



$$\hat{\rho}(x) \approx \sum_{ij} \phi_i(x) \text{Im} \left( \sum_{l=1}^P \frac{\omega_l}{H - (z_l + \mu)S} \right) \phi_j(x)$$

- (quasi-)1D:  $\mathcal{O}(N)$
- (quasi-)2D:  $\mathcal{O}(N^{3/2})$
- 3D:  $\mathcal{O}(N^2)$

- Relatively small number of poles (20-30)
- Trivially parallelizable over poles
- Only selected elements of inverse are needed

(Due to sparsity of the target density matrix)

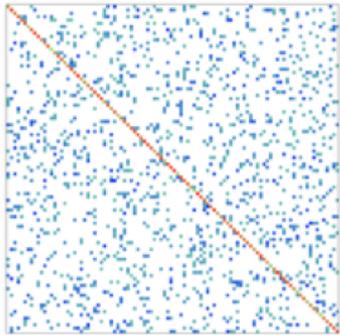
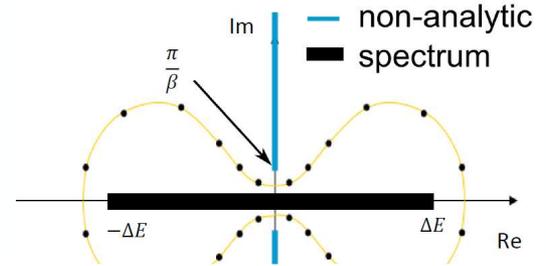
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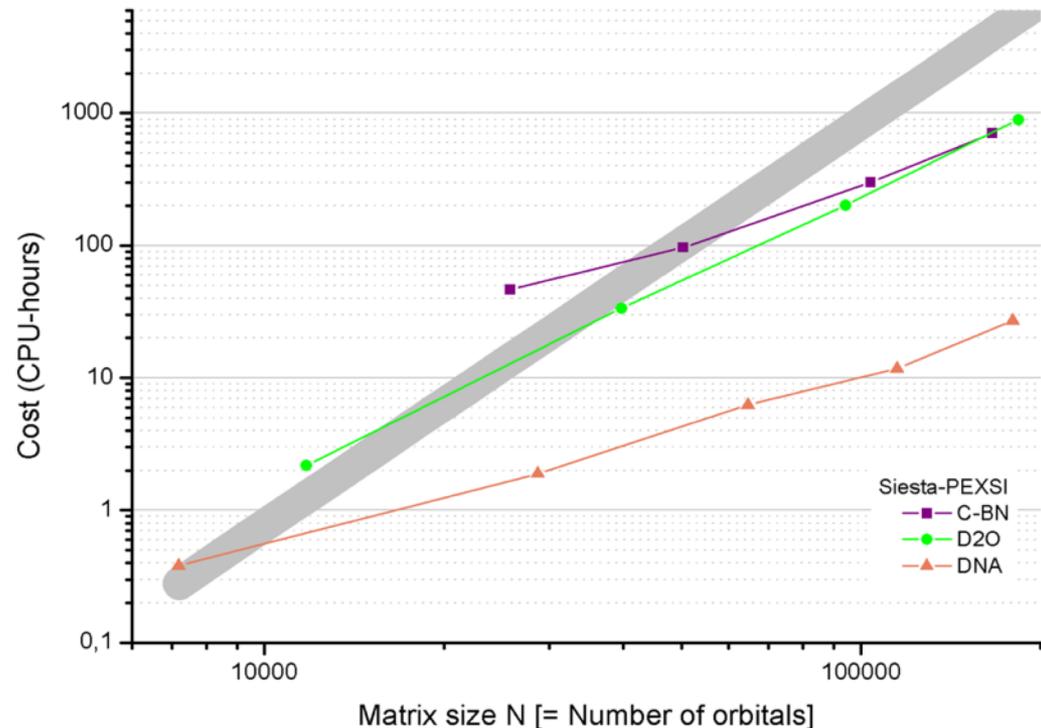
$$\hat{\rho}(x) \approx \sum_{ij} \phi_i(x)$$

(quasi-)1D:  $\mathcal{O}(N)$

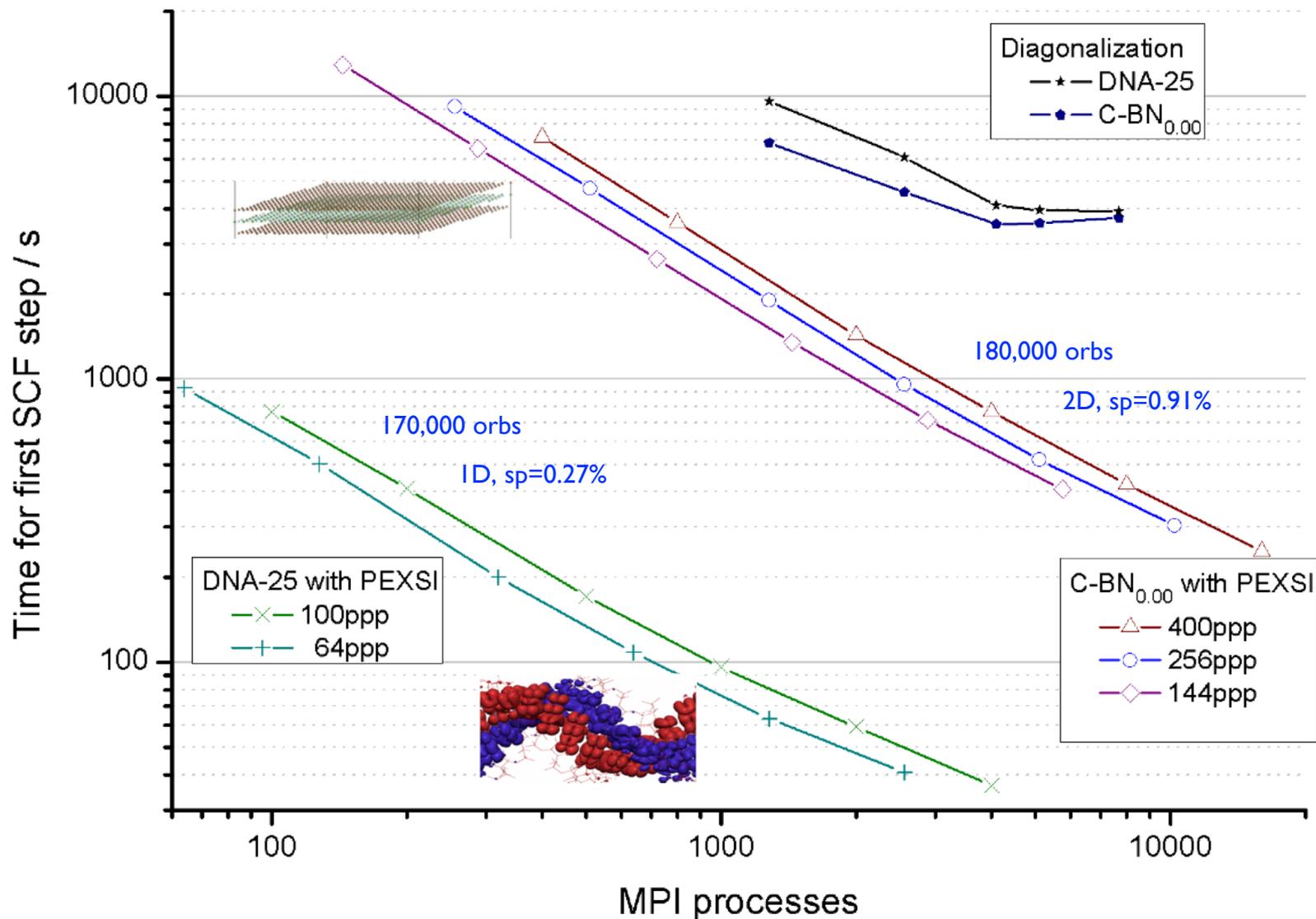
(quasi-)2D:  $\mathcal{O}(N^{3/2})$

3D:  $\mathcal{O}(N^2)$

(Due to sparsity of the target density

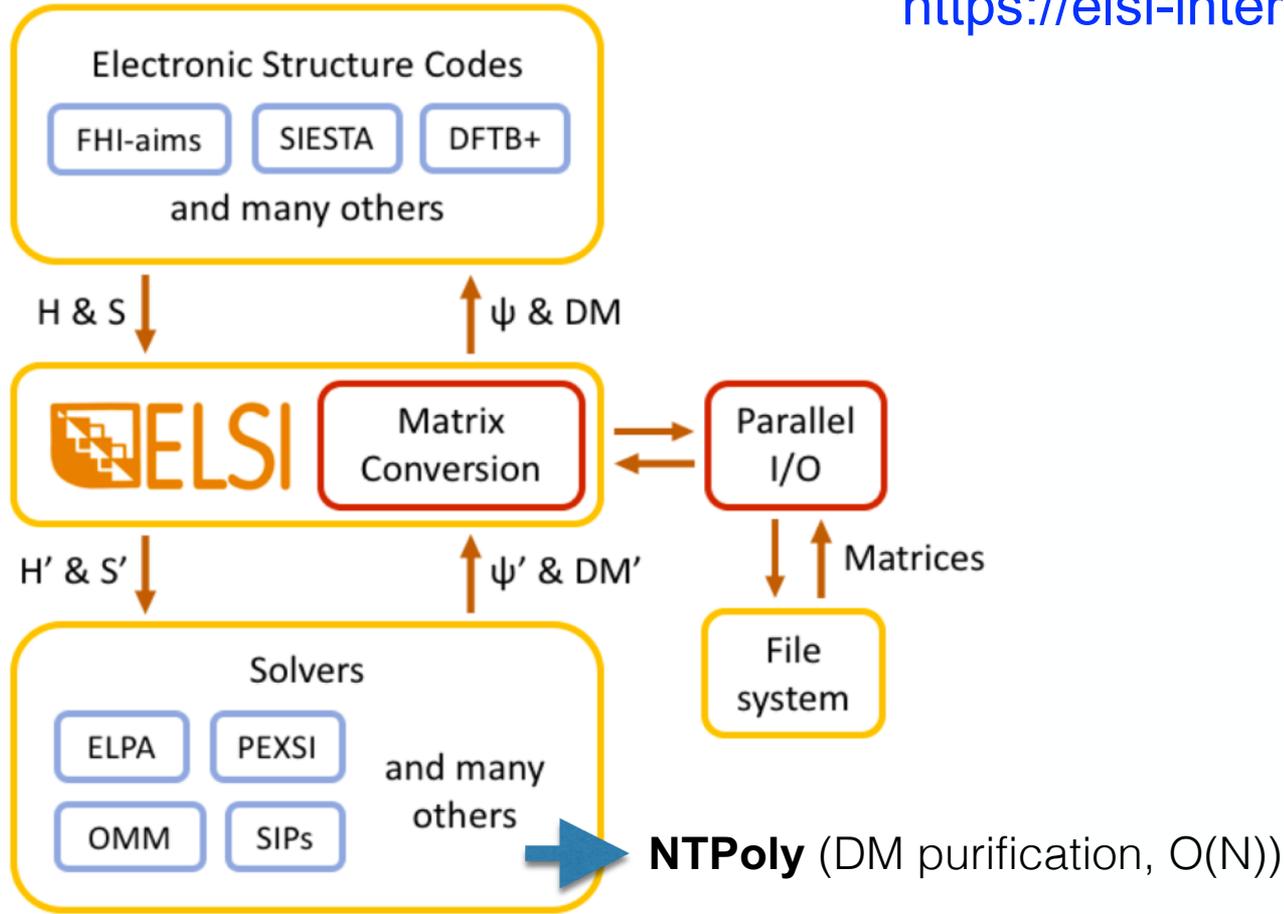


# Massive scalability: PEXSI solver



# ELSI initiative to integrate solver libraries

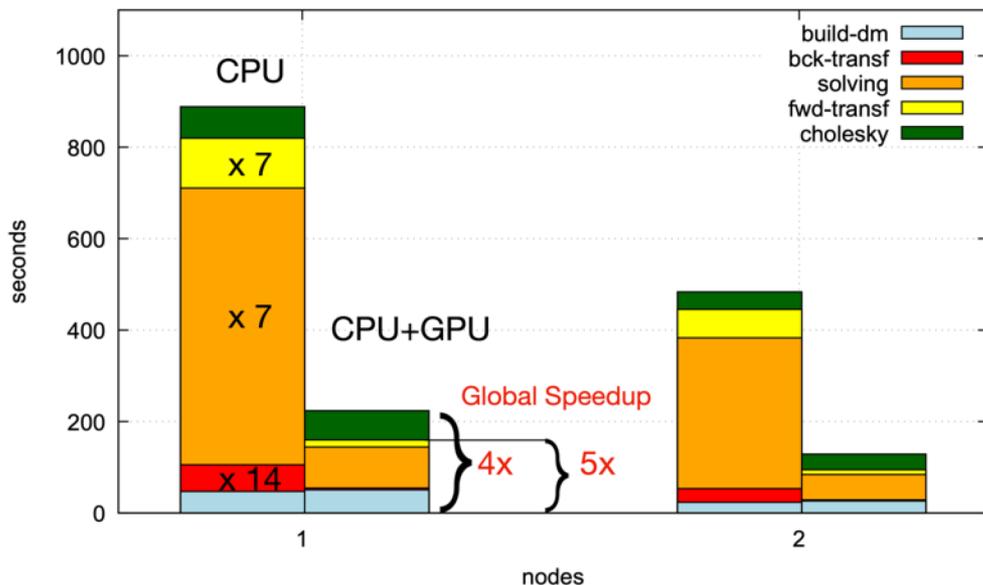
<https://elsi-interchange.org>



# GPU acceleration for diagonalization

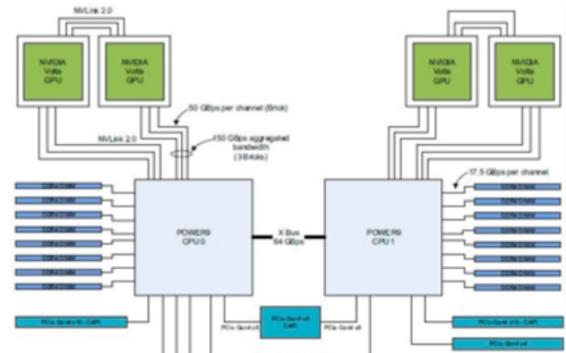
Marconi-100 (CINECA): 32 CPUs+ 4 GPUs /node

## ELSI-ELPA GPU acceleration

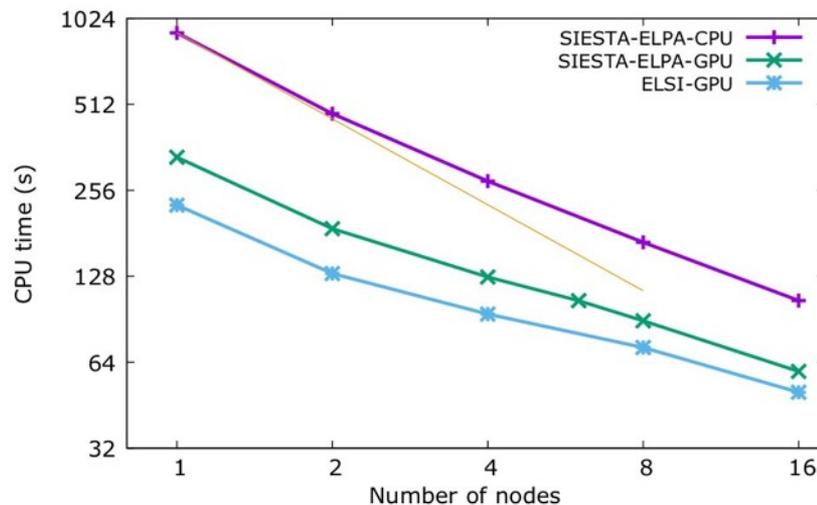


Future enhancements in ELPA (better kernels) and in ELSI (e.g. **build-DM** stage) are integrated in SIESTA automatically

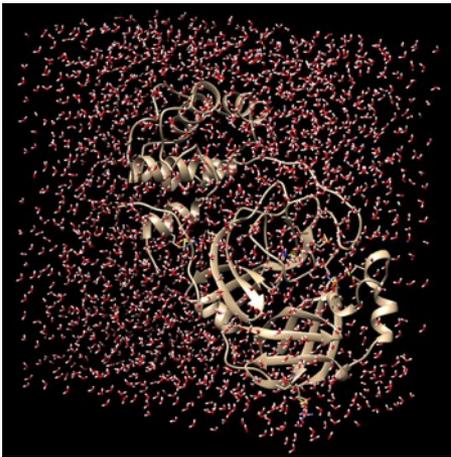
System: Si quantum dot, with approx 35000 orbs



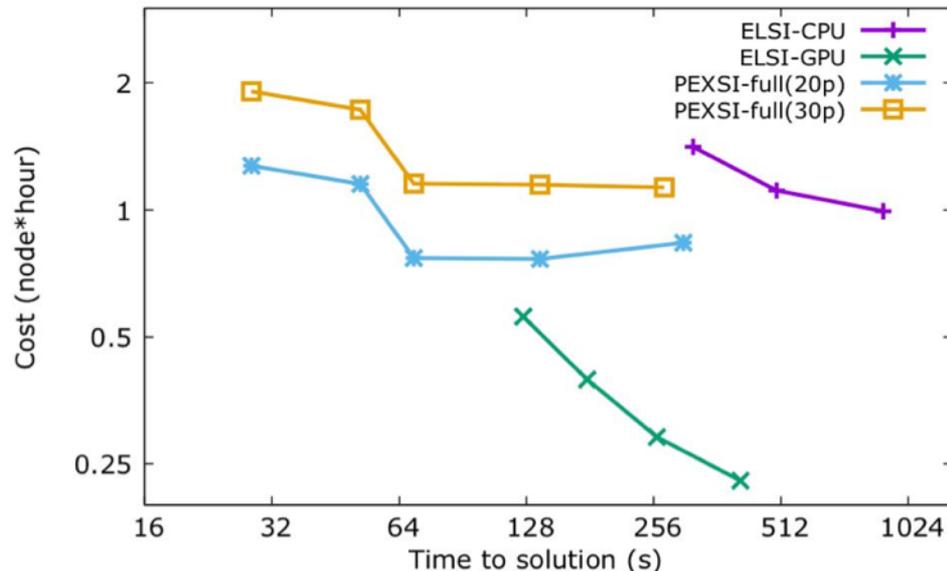
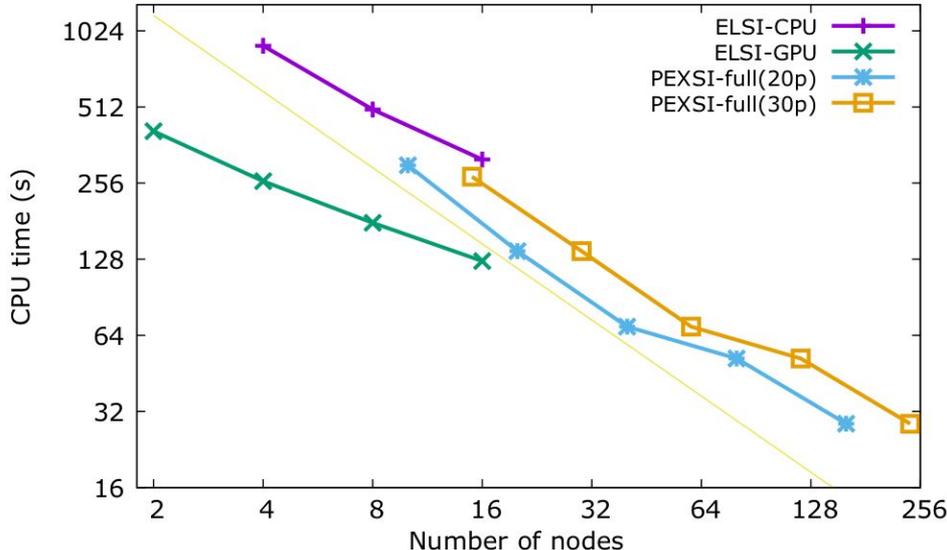
## Proper binding of GPUs to MPI ranks



# Comparison of global efficiency of solvers for a very large problem



SARS CoV-2 M<sup>pro</sup> with solvation water molecules  
 Approx. 8800 atoms; 58000 orbitals



Work on GPU acceleration of PEXSI library is under way



DRIVING THE EXASCALE TRANSITION

THANKS

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